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Corrigendum
Higher-dimensional analogs of Châtelet surfaces
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Let K/k be a cyclic Galois extension of fields of degree n, and let $P(x) \in k[x]$ be a separable polynomial of degree dn or $dn-1$. Let X_0 be the affine norm hypersurface in $\mathbb{A}_k^{n+1}$ given by

$$N_{K/k}(\vec{z}) = P(x) \neq 0. \tag{1}$$

In [**1**, § 2], we attempted to construct a smooth proper model X of X_0 extending the map $X_0 \to \mathbb{A}_k^1 \setminus V(P(x))$ given by $(\vec{z}, x) \mapsto x$ to a map $X \to \mathbb{P}_k^1$. However, [**1**, Proposition 2.1] is false whenever $n > 2$. In this note, we explain how all statements and proofs of [**1**] can be rectified using a construction from [**2**], particularly the following theorem.

Theorem 1 [**2**, Theorem 1]. *Let K/k be a cyclic Galois extension of fields of degree n, and let $P(x) \in k[x]$ be a separable polynomial of degree dn or $dn-1$. There exists a smooth proper compactification X of X_0, fibered over $\mathbb{P}_k^1 = \operatorname{Proj} k[x_0, x_1]$, such that $X \to \mathbb{P}_k^1$ extends the map $X_0 \to \mathbb{A}_k^1$. Furthermore, the generic fiber of $X \to \mathbb{P}_k^1$ is a Severi–Brauer variety, and the degenerate fibers lie over $V(P(x_0/x_1)x_1^{dn})$, and consist of the union of n rational varieties all conjugate under* $\operatorname{Gal}(K/k)$.

Replacing the variety X of [**1**, § 2] with the variety X from Theorem 1 immediately rectifies all but one of the statements and proofs of [**1**, §§ 3–5] (we return to this exception below). More precisely, the proofs of [**1**, Proposition 3.1 and Theorem 3.2] depend only on

(1) the generic fiber of $X \to \mathbb{P}^1$ being a Severi–Brauer variety and
(2) the degenerate fibers of $X \to \mathbb{P}^1$ lying over $V(P(x_0/x_1)x_1^{dn})$ and consisting of the union of n rational varieties, all conjugate under $\operatorname{Gal}(K/k)$.

That these properties hold is exactly the result of Theorem 1. The remaining proofs of statements in [**1**, §§ 4–5] rely on [**1**, Proposition 3.1 and Theorem 3.2] without further mention of the specific geometry of the fibration $X \to \mathbb{P}^1$, except for part of the proof of [**1**, Theorem 1.3].

To rectify the remaining part of the proof of [**1**, Theorem 1.3], we must correct the construction of the Châtelet p-fold bundle over $\mathbb{P}^1 \times \mathbb{P}^1$ given by $u^p P_\infty(x) + P_0(x)$. To do so, we carry out the same constructions as in [**2**] over the polynomial rings $k[u, x]$, $k[u^{-1}, x]$, $k[u, x^{-1}]$, and $k[u^{-1}, x^{-1}]$ and glue to construct a bundle over $\mathbb{P}^1 \times \mathbb{P}^1$. More precisely, the proof of [**1**, Theorem 1.3] requires a smooth compactification of the normic bundle $X_0 \to U \subset \mathbb{A}^2$ given by

$$N_{K/k}(\vec{z}) = u^p P_\infty(x) + P_0(x) \neq 0,$$

where the closure of $V(u^p P_\infty(x) + P_0(x))$ in $\mathbb{P}_u^1 \times \mathbb{P}_x^1$ is smooth and of bidegree $(d_1 p, d_2 p)$ for some positive integers d_1 and d_2. (In characteristic p, we instead consider the normic bundle X_0 given by $N_{K/k}(\vec{z}) = u^p P_\infty(x) + u^{p-1} P_\infty(x) + P_0(x) \neq 0$, with the same conditions on the closure of the curve in $\mathbb{P}_u^1 \times \mathbb{P}_x^1$.) Sections 3 and 4 of [**2**] give a smooth partial compactification

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$X \to \operatorname{Spec} R$ of any normic bundle $X_0 \to D(a) \subset \operatorname{Spec} R$ given by $N_{K/k}(\vec{z}) = a \neq 0$ for any k-algebra R, as long as $V(a)$ is smooth in $\operatorname{Spec} R$. Thus, we may apply [**2**, §§ 3 and 4] to construct relative compactifications over the polynomial rings $k[u, x]$, $k[u^{-1}, x]$, $k[u, x^{-1}]$, and $k[u^{-1}, x^{-1}]$. Then, since the closure of $V(u^p P_\infty(x) + P_0(x))$ (respectively, $V(u^p P_\infty(x) + u^{p-1} P_\infty(x) + P_0(x))$) has bidegree $(d_1 p, d_2 p)$ for some positive integers d_1 and d_2, we may glue the relative compactifications as in [**2**, Lemma 4] to construct a smooth proper model $X \to \mathbb{P}^1 \times \mathbb{P}^1$.

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